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Supercomputer simulation of a gravitating gaseous circumstellar disk using SPH*

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Abstract. New parallel algorithm is developed for simulating the dynamics of a thin circumstellar disk. It is based on the combination of a gridless method of smoothed particle hydrodynamics (SPH) and the grid-based convolution method for calculating a gravitational potential. To develop the algorithm, we made software profiling of numerical experiments aimed at simulating gravitational fragmentation and subsequent inward migration of dense clumps onto the star.

1. Introduction

One of important problems of the modern computational astrophysics is the development of numerical models for the simulation of circumstellar disks and formation of planets. In the last decade a great body of observational data appeared [1], and these data require theoretical interpretation and explanations with the help of numerical experiments. Typically these numerical simulations need a high resolution to track fragmentation of a disk into clumps and their subsequent migration.

In this regard the gridless method of smoothed particle hydrodynamics [2] (SPH) has some advantages over grid methods: it allows treating rapid changes in the density at a very small spatial scale. On the other hand, a conventional implementation of a gravitational solver used in conjunction with the SPH makes possible to use the tree-code algorithms [3], which have certain drawbacks in parallel implementation in comparison with the grid-based Poisson solvers.

In recent papers [4, 5], we have developed a numerical method that combines the gridless SPH to solve gas-dynamic equations with the grid-based method for calculating a gravitational potential. In the present paper we study the validity and efficiency of the method proposed by focusing on the problem of simulating the episodic accretion of gas clumps from a circumstellar disk onto a protostar [6]. Based on the parallel method proposed in [7, 8] and using the profiling results of the numerical experiments we have developed a parallel algorithm that is suitable for general purpose simulations of gravitating gaseous disks.

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2. A numerical model of thin gaseous circumstellar disk

Here we give a general description of the model, while the details can be found in [5]. The mathematical model consists of gas-dynamic equations for the surface density and the three-dimensional Poisson equation:

$$\sigma_{\rm gas} = \int_{-\infty}^{+\infty} \rho_{\rm gas}\, dz, \quad p^* = \int_{-\infty}^{+\infty} p\, dz,$$
$$\frac{\partial \sigma}{\partial t} + {\rm div}(\sigma \boldsymbol{v}) = 0, \quad \sigma \frac{\partial \boldsymbol{v}}{\partial t} + \sigma(\boldsymbol{v}, \nabla)\boldsymbol{v} = -\nabla p^* - \sigma \nabla \Phi,$$
$$\frac{\partial S^*}{\partial t} + (\boldsymbol{v}, \nabla) S^* = 0, \quad p^* = T^* \sigma.$$

Here $\boldsymbol{v}$ is the gas velocity in the disk plane, p^* is the surface gas pressure, γ^* is the polytrope index for quasi-3D, which is related to poytrope index γ by $\gamma^* = 3 - \frac{2}{\gamma}$, $T^* = \frac{p^*}{\sigma}$, $S^* = \ln \frac{T^*}{\sigma^{\gamma^*-1}}$ are the derivative values analogous to gas temperature and entropy, $\boldsymbol{a} = -\nabla\Phi$ is the particles accelleration in the external and self-consistent gravitation field, Φ is the gravitational potential, which is the sum of a central body potential and a disk potential, $\Phi = \Phi_1 + \Phi_2$, $\Phi_1 = -\frac{M_c}{r}$, M_c is the central body mass, Φ_2 is the self-consistent gravitational field calculated from the Poisson equation

$$\Delta\Phi_2 = 4\pi\sigma_{\rm gas}, \quad \Phi_2 \to 0 \text{ at } r \to \infty.$$

Equations are given in the dimensionless form. The base parameters are the gravitational constant G, the typical size $R_0 = 10\text{AU} = 1.5 \cdot 10^{12}$ m (AU, astronomical unit), and the typical mass $M_\odot = 2 \cdot 10^{30}$ kg.

Initial parameters of the disk are the inner radius $R_{\rm min}$, the external radius $R_{\rm max}$, and the initial values of temperature and surface density taken from [9]. The surface density is defined as $\Sigma = \Sigma_0/r$, where Σ_0 is computed from the equation $\int_{R_{\rm min}}^{R_{\rm max}} \Sigma_0\, dr\, d\phi = M_{\rm disc}$, and the gas temperature is $T = T_0/\sqrt{r}$, where T_0 is a user-defined parameter.

The gas velocity is calculated using the equation of gas rotation around the central body:

$$\frac{v_\phi^2}{r} = \frac{1}{\Sigma}\frac{\partial p^*}{\partial r} + \frac{\partial \Phi}{\partial r}, \quad v_r = 0.$$

The numerical algorithm is based on a combination of the SPH for solving gas-dynamic equations with the grid-based Poisson solver: the surface density is interpolated into the nodes of a Cartesian grid and then is used for calculating a gravitational potential using the convolution method.

3. Numerical experiments

The main purpose of the experiments performed is to test the numerical method by simulating the fragmentation of a gaseous disk into clumps and tracking their evolution up to the moments when the so-called episodic accretion may occur. Then it could be possible to make a comparison with the numerical results known from literature (and obtained with other numerical methods) and to make up a basis for the software profiling and subsequent development of a new scalable parallel algorithm.

The initial size of a circumstellar disk is set to be from 10 to 100 AU. The mass of a protostar is equal to $0.8M_{\mathrm{Solar}}$, the temperature is 90 K on the inner radius of the disk at 10 AU, and 30 K on the outer radius at 100 AU. The SPH particles are coming closer to the protostar than a user-defined parameter R_{cell} (the radius of a "sink cell"), were considered as being consumed by the protostar.

The number of particles varied from 160,000 to 640,000. In most experiments we took the grid size to be equal to 512×512. To verify the calculations we used a finer grid with the number of nodes $1{,}024 \times 1{,}024$ and $2{,}048 \times 2{,}048$. The initial value of an adaptive timestep was set to be 0.03 year, and then it could be decreased by the factor of 2 or 4 to calculate the dynamics of clumps in the proximity of the protostar.

The accretion rate is calculated using the star mass M_c by the formula: $\dot{M} = \frac{M_c(t+\tau) - M_c(\tau)}{\tau}$. Here the timestep τ varies from 3 to 120 years, a particular value τ is specified in the description of figures.

Most supercomputer experiments were conducted on NKS-30T in the Siberian Supercomputing Center, while the remaining part — on the supercomputer "Lomonosov" in the Moscow State University.

For automatic tracking of gas clumps (Figure 1) we have also implemented a special algorithm of seeking, tracking clumps (local peaks of the surface density), and retrieving their main characteristics (like velocity and mass).

3.1. The accretion rate in fragmenting and non-fragmenting disks. As the first step we have checked the ability of the numerical model to reproduce different modes of the accretion rates $\dot{M}$ for fragmenting and non-fragmenting disks. We have carried out several experiments for the disks with masses 0.15, 0.2 and 0.25 M_{Solar} and compared their accretion rates (Figure 2). The left, the central and the right part of Figure 2 present the results of calculations with different timestep and grid parameters . The main features of the numerical solutions are the same for all calculations.

As was expected, non-fragmenting disks (with masses 0.15, 0.2) have demonstrated smooth accretion rates in the limits $5 \cdot 10^{-7}$–$5 \cdot 10^{-6}$ solar masses per year. If we increase the mass of a disk from 0.15 to 0.2 M_{Solar}, the accretion rate will increase by the factor of 1.3.

On the other hand, the model of a fragmenting disk with the mass $0.25\,M_{\mathrm{Solar}}$ shows the development of a non-smooth accretion rate with

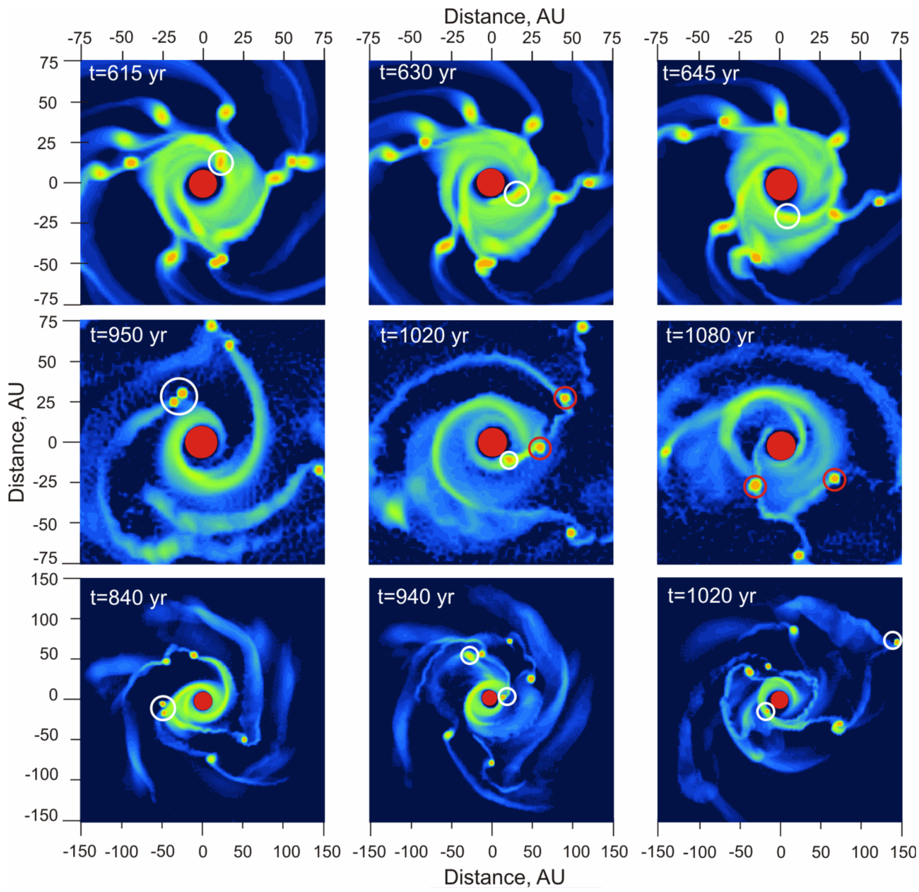

Figure 1. Example of computations which require tracking of clumps

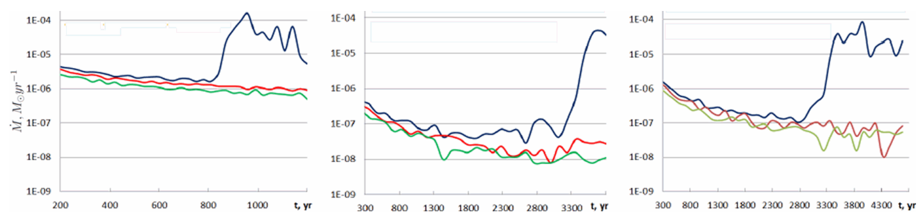

Figure 2. The accretion rate for models with the disk mass equal to 0.15 (the green line, a non-fragmenting disk), 0.2 (the red line, a non-fragmenting disk) and 0.25 M_{Solar} (the blue line, a fragmenting disk). The timestep in years and the grid are 30 and 512×512 (left), 120 and 1024×1024 (center), and 120 and 2048×2048 (right)

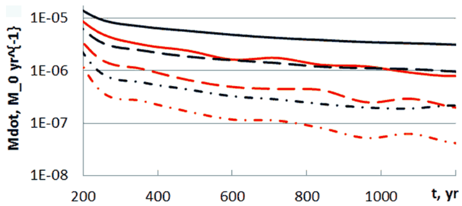

Figure 3. The accretion rates for a disk with the mass 0.2 M_{Solar}. The black and red lines correspond to different types of a softening kernel. The solid lines correspond to the calculation with 4,0000 particles, the dotted lines—160,000 particles, and the dash-dotted lines—640,000 particles

episodic outbursts after 850 years of its evolution. The results obtained are in agreement with those in [10, 11].

Figure 3 shows diagrams of the accretion rate for a disk with the mass 0.2 M_{Solar} for different parameters. It is clear that the absolute value of the background accretion rate is sensitive to the numerical resolution. If we increase the number of SPH-particles from 40,000 (the solid line) to 640,000 (the dashed-dotted line), the accretion rate will decrease for more than 1 order of magnitude. A similar decay of the accretion rate can be observed for models with different types of SPH softening length (the black and the red lines in Figure 3). These results confirm the idea that the viscosity terms, related to the numerical viscosity play a significant role in reproduction of the backgrount accretion rate.

The results presented in Figures 2 and 3 can be interpreted in the following way: the developed numerical model is able to reproduce different scenarios of the protostar accretion with a strong dependence among dynamic processes in the disk (the clumps formation and their mutual interaction) and the rate of mass transfer onto the protostar.

3.2. The clumps-induced outburst mode of episodic accretion. Another interesting result of the numerical experiments is the outburst mode of the accretion, related to the gravitational fragmentation of a disk. Such a scenario was earlier described in [11], where it was demonstrated that the clump rotating around the protostar may disturb the inner part of a disk and initiate the accretion onto the protostar. Figure 4 shows the surface density of gas, obtained for the model with 0.25 M_{Solar} for the four different stages of evolution. Clumps, which induce the accretion outbursts, are marked with the white circles.

The accretion rate is constantly increasing during several hundreds years and then it happens to increase rapidly from the rate $5 \cdot 10^{-6}$ to $4.5 \cdot 10^{-4}$ of the solar masses per year at the moment ~ 800 years. During the subsequent evolution of the disk, the accretion rate changes non-smoothly: it has several peaks of a smaller amplitude and then it returns back to the “pre-outbursts” values. Figure 4 shows that the marked clump initially rotates around the protostar on the radius about 50 AU, its mass being about 10 M_{Jupiter}. The

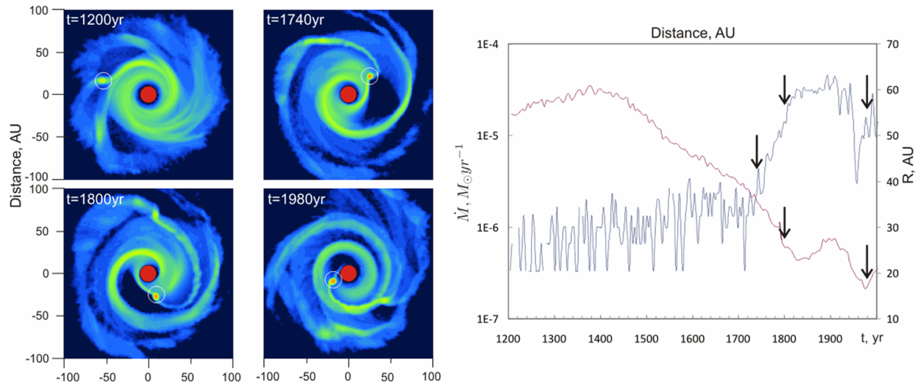

Figure 4. Left: the logarithm of gas surface density of the disk evolution, obtained in 1200, 1740, 1800, 1980. The disk mass is 0.25 M_{Solar}. Right: the accretion rate, measured in $M_{\odot}y^{-1}$, for the same period (the solid line), the orbital radius of the marked clump (the dashed line). The time step is equal to 30 years. Moments of figures capturing are marked with arrows

first accretion outburst occurs when the clump begins inward migration and reaches proximity of the protostar. However, it never falls down the protostar. Instead, it continues the rotation around the protostar at distances of 15–20 AU.

To check whether this behavior depends on the position of an internal sink cell we have carried down computations with decreasing the radius of a sink cell from 10 to 5 AU. A comparison between the accretion rates and the orbital position of the nearest clump is shown in Figure 5. The clump is slowly approaching the protostar and initiates several accretion outbursts while moving to the inner part of the disk. This scenario suggests that under certain conditions the astronomically observed accretion outbursts may give indirect evidence of the existence of clumps in the inner part of a disk.

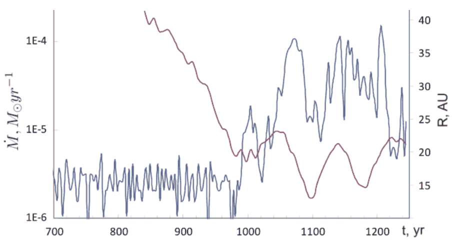

Figure 5. The blue line — the accretion rate for a disk with the mass 0.25 M_{Solar} in units of $M_{\odot}$ y^{-1}, the radius of the sink cell $R_{\mathrm{cell}} = 5$ AU. The red line— the orbital radius of the clump nearest to the protostar. The time step for calculating the accretion rate is 3.75 years

4. The parallel algorithm

Based on the numerical experiments performed using a very simple parallel algorithm [12], we made the performance analysis and profiling in order to define the bottlenecks and the key properties of the numerical method to focus on creating its parallel version. The developed parallel algorithm uses the domain decomposition to N uniform domains. Each subdomain has its own group of processors: the main processors are used to calculate the Poisson equation while other processors (CPU/GPU) are used to process SPH-particles. The dynamic load balancing is used for re-distributing the processors among groups according to the number of SPH-particles distributed among subdomains.

The general scheme of the algorithm was proposed in [7, 8] and, in addition, we introduce the following key ideas and modifications:

- The amount of computations required for solving gas-dynamic equations is much higher (by the factor of 100 and even more) than for solving the gravitational potential. This means that the main computational efforts should also be concentrated on the SPH part.
- For modern supercomputers the amount of computations required for processing one SPH particle (calculating its coordinates and gas parameters) is much higher than for transferring this particle from one processor to another. Thus, there is no point in trying to avoid communications as it used to be several years before.
- For a single time step the SPH particle can move among nearest cells, only. Because of this fact the number of particles which needed to be transferred among subdomains does not exceed 0.1–1 % of the total number of the SPH-particles.
- Processing of the SPH particles requires the neighboring particles to be located on the same processor. This requires the use of the so-called "ghost" zones or overlapping zones among subdomains: when the information about particles located near the boundaries of subdomains is distributed in both subdomains.
- We use sorting for the SPH-particles and re-distributing them among subdomains.
- To optimize the search for the neighbors, we employ constructing K-d trees inside each cell in the case if the number of the SPH particles is big enough.

The first proof-of-concept experiments with the parallel algorithm have shown that it is promising for the use on supercomputers with a moderate number of processors (100–1,000), with the number of the SPH particles 100–1,000 million and the size of a grid up to $32{,}768 \times 32{,}768$. For a typical numerical experiment with 100 million of the SPH particles, $16{,}384 \times 16{,}384$ grid nodes, and using 128 CPU-cores, the calculation of a single time step is about 20 seconds, with less than half computations spent on communications.

5. Conclusion

We have presented results of the numerical experiments aimed at the simulation of the episodic accretion in a circumstellar disk. It was shown that the earlier created numerical model, which is a combination of gridless SPH method and the grid-based Poisson solver, is valid and can be used for the simulations of a gaseous gravitating disk characterized by the fragmentation into clumps. We have developed a parallel algorithm, based on the domain decomposition and dynamic load balancing which is promising for conducting numerical experiments with a big number of particles and a fine grid.

References

[1] Howard A.W. Observed properties of extrasolar planets // Science. — 2013. — Vol. 340. — P. 572–576.

[2] Gingold R.A., Monaghan J.J. Smoothed particle hydrodynamics. Theory and application to non-spherical stars // Monthly Notices of the Royal Astronomical Society. — 1977. — Vol. 181. — P. 375–389.

[3] Barnes J.E., Hut P. A hierarchical $O(N \log N)$ force-calculation algorithm // Nature. — 1986. — Vol. 324. — P. 446–449.

[4] Stoyanovskaya O.P., Snytnikov V.N. Clump formation due to the gravitational instability of a multiphase medium in a massive protoplanetary disc // Mon. Not. R. Astron. Soc. — 2013. — Vol. 428. — P. 2–12.

[5] Stoyanovskaya O.P., Snytnikov N.V., Snytnikov V.N. An algorithm for solving transient problems of gravitational gas dynamics: a combination of the SPH method with a grid method of gravitational potential computation // Computational Technologies and Programming. — 2015. — Vol. 16. — P. 52–60 (In Russian).

[6] Audard M., Abraham P., Dunham M.M., et al. Episodic accretion in young stars // Protostar and Planets VI / H. Beuther, R. Klessen, C. Dullemond, T. Henning, eds. — 2014. — P. 387–410.

[7] Snytnikov N.V., Stoyanovskaya O.P. domain decomposition algorithm using SPH and PIC methods for simulating gas-dust gravitating disks // Bull. Novosibirsk Comp. Center. Ser. Num. An. — Novosibirsk, 2015. — Iss. 17. — P. 69–78.

[8] Snytnikov N.V. Scalable parallel algorithm for simulation of 3D dynamics of gravitating systems using particle-in-cell method // Proc. Int. Conf. "Parallel Computational Technologies (PCT'2016)", 2016. — P. 298–305.

[9] Vorobyov E.I. Embedded protostellar disks around (sub-) solar protostars. I. Disk structure and evolution // ApJ. — 2010. — Vol. 723. — P. 1294–1307.

[10] Vorobyov E. I., Basu S. Mass accretion rates in self-regulated disks of T Tauri stars // ApJ. — 2008. — Vol. 676. — P. 139–142.

[11] Machida M., Inutsuka S., Matsumoto T. Recurrent planet formation and intermittent protostellar outflows induced by episodic mass accretion // ApJ. — 2011. — Vol. 729. — P. 1–17.

[12] Stoyanovskaya O.P., Snytnikov V.N. Solving non-stationary problems of two-phase gravitating matter using supercomputers: problems and results // Proc. Int. Conf. "Parallel Computational Technologies (PCT'2012)". — 2012. — P. 683–689.